

Electron acceleration due to photon absorption: A possible origin of the infinity problems in relativistic quantum fields

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Based on the concept of extended particles recently introduced we perform a Gedankenexperiment accelerating single electrons with photons of suitably low frequency. Accounting for relativistic time dilation due to the acquired velocity and in infinite repetition of single absorption processes it can be shown that the kinetic energy in the infinite limit is equal to $m_e c^2/2$. However, the inertial mass of the electron seems enhanced, and it can be established that this enhancement is described by the relativistic mass effect. It appears, therefore, that although there exists a singularity in interactions - the frequency required to accelerate the particle near the limit of c becomes infinite - the energy of the particle itself approaches a finite limit. Comparing with calculations of the Lamb-shift by Bethe this result seems to provide the ultimate justification for the renormalization procedures employed.

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A realistic interpretation of quantum theory (QT), or the interpretation of the wave function as a real, i.e. *physical* wave, which was Schrödinger's original concept [1], has always been contradicted by compelling evidence. Currently there exist four major obstacles for a realistic reformulation of QT: (i) The uncertainty relations [2], (ii) the dispersion relations for massive particles like electrons [3], (iii) intrinsic energy components due to electrostatic interactions, and (iv) the experimental proofs against local and realistic theories [4]. The theoretical proofs against these theories by von Neumann or Jauch and Piron [5,6] are not as convincing, since they leave quite a few loopholes, as Bohm and Bell pointed out [7,8].

Due to this evidence the search for an interpretation of microphysics in terms of *extended particles* remained something of a minority program: although some efforts have been made, they always encountered unsurmountable difficulties [9].

As recently established, these difficulties could be due to an only restricted analysis of the fundamental relations in QT: this type of analysis, called the *formal approach* proceeds from the Schrödinger relation and/or the commutation relations, which are accepted without limits [10]. Using a *realistic* approach, the result changes drastically. If wave-features of single particles are interpreted as physical waves describing the intrinsic structure due to particle propagation, the fundamental axioms of QT gain the following meaning: (i) Due to an additional and intrinsic energy component, the dispersion relations of monochromatic waves are valid also for massive particles, (ii) the Schrödinger equation is, also due to this intrinsic energy component, no longer an exact equation, (iii) the uncertainty relations remain valid, but they are no longer axioms, but rather the error margin due to the omission of intrinsic energy in Schrödinger's equation. Especially the latter point seemed interesting, because it allows, in principle, to describe processes without a limit of precision also at the microphysical level.

The last two problems to a realistic interpretation were

removed by establishing, that also the Maxwell equations [11] are a description of intrinsic particle properties, and while QT is mainly concerned with longitudinal wave properties, electrodynamics (ED) describes the transversal and intrinsic fields of propagation. In this case it can be deduced that electrostatic fields of interaction are a consequence of photon exchange, which removes obstacle (iii), since these interactions vanish for a particle in constant motion. In addition, the concept of spin in QT could be referred to the transversal and intrinsic magnetic fields of particle propagation: since in the realistic picture of extended particles spin is an oscillating variable, a valid measurement of spin correlations requires a local precision explicitly higher than allowed by the uncertainty relations: in this case the measurement cannot be described consistently within the framework of QT, the experimental results (iv) therefore allow no longer the conclusion, that a local and realistic theory is inconsistent. In view of these results it seems that the approach provides an alternative to the standard interpretation, and which can be used to describe fundamental processes on a micro level.

One of the most intriguing consequences of Einstein's theory of Special Relativity (STR) [12,13] is the energy relation for a mass m in a state of motion u [14,15].

$$E = \sqrt{m^2 c^4 + \mathbf{p}^2 c^2} = m \gamma c^2 \quad \gamma = \frac{1}{\sqrt{1 - u^2/c^2}} \quad (1)$$

In the small velocity range, commonly identified as the non-relativistic range, the relation reduces to:

$$E(u \ll c) = mc^2 + \frac{1}{2} m u^2 \quad (2)$$

This expression is, as Einstein pointed out [16], equal to the classical expression in Newtonian mechanics with an additional term, the rest energy of m .

As recently established, the kinetic energy of an electron is only half of its total energy, the correction results

from intrinsic energy components due to wave-like internal properties [10]. The result was obtained in a reference frame at rest, and it could also be derived that electrostatic interactions correspond to photon emission and absorption processes. The energy of a particle after absorption of a photon is given by:

$$E_1 = E_0 + E_{ph} = mu_0^2 + m_{ph}c^2 \quad (3)$$

where m_{ph} denotes the mass of the absorbed photon. That photons or light pulses also transfer mass $m_{ph} = E_{ph}/c^2$ was demonstrated by Einstein in a Gedankenexperiment in 1906 [16]. The connection of photon mass with the rest energy term in STR is, also for a non-moving inertial frame, a rather obvious one: if the whole energy of a body is converted into electromagnetic energy, its mass shows up as mass of the emitted photons.

Apart from this new interpretation, the result is no change of established concepts and fairly standard. But as this result is derived within a non-relativistic framework, it is somewhat beyond Einstein's original derivation, and it raises, for this reason, a new and interesting question. In this case the problem becomes imminent, whether the relativistic energy for moving electrons and its singularity at $u \rightarrow c$ do in every case signify a real, i.e. *physical* effect, or whether they cannot, in some cases, be considered more or less *virtual*. Virtual meaning, that the effect only is a way to account for energy transfer in electromagnetic fields. This question has been addressed to some extent (see, for example, Adler's paper ref. [14]), and the common understanding seems to be, that "the time kept by a rapidly moving particle is dilated and hence, as the particle's speed increases, apparently greater time intervals are taken to produce the same effect, thus the apparent increase in resistance." But in this case the same reasoning can be applied to the energy, and it must be analyzed, whether its singularity is not, equally, a result of changed interaction characteristics. To decide on this question we perform a Gedankenexperiment with photons of (suitably) low frequency, and calculate the acceleration of the electron due to photon absorption numerically.

The paper is organized as follows: first the one particle problem will be treated in a frame at rest, and it is assumed, that the photon source is in no other way interacting with the particle. The velocity and energy of the particle due to photon absorption will be calculated and compared with the relativistic expression. It will be shown, that the kinetic energy of a particle cannot exceed the limit given by $mc^2/2$. As this result suggests a thorough analysis of the interaction process, we will then assume that the classical and non-relativistic accelerations are due to electrostatic fields. Comparing the classical changes of velocity to the changes pertaining to the photon-absorption model, it can be established that the relativistic mass effect - or the enhancement of inertia - must be attributed to the time dilation in the electron system. The result is compared to existing measurements

of the ratio $(e/m) = (e/m)(u)$ [17–24], and it is shown that the interpretation is in accordance with experimental evidence. Relating the derived results on electron energy to the prevailing infinity problems in quantum electrodynamics and their proposed solution via renormalization procedures [25]–[35] it can be shown that these infinity problems typically arise from the singular properties of energy in relativity theory.

The electron with mass m shall initially be at rest. The photon source with frequency ω_0 in our inertial frame, where ω_0 shall be very small in order to yield differential-like accelerations, is also at rest in the same frame S_0 . We consider only linear accelerations, the system is therefore limited to one dimension, say x , with the initial position of the particle $x_0 = 0$. As the acceleration during interaction depends on the differential of energy density, as recently demonstrated [10], we shall not consider the acceleration process but merely evaluate the velocity before and after absorption.

The only effect to be considered in the one-particle problem is the relativistic time dilation due to relative motion of the particle. The energy in absorption processes depends on the eigenzeit in the system of the particle [10]. Due to time dilation the absorbed energy will be diminished, energy and velocity of the particle after n photon absorptions are therefore given by:

$$\begin{aligned} E_n &= \hbar\omega_0 \left[1 + \sum_{i=0}^{n-1} \sqrt{1 - \left(\frac{u_i}{c}\right)^2} \right] = m u_n^2 \\ u_n &= \sqrt{\frac{\hbar\omega_0}{m}} \left[1 + \sum_{i=0}^{n-1} \sqrt{1 - \left(\frac{u_i}{c}\right)^2} \right]^{1/2} \end{aligned} \quad (4)$$

To compare with Einstein's expression we add the rest energy and take only half of the total energy of the particle. Performing the interactions with a suitably chosen photon frequency in a numerical calculation, it can be seen that the kinetic energy of the particle is equal to Einstein's expression only in the low velocity range, while in the high range the energy converges to a limit of $1.5mc^2$ in the interaction model, whereas it becomes irregular in the STR model (Fig. 1).

This result for the energy of the particle suggests a comparison with accelerations in electrostatic fields, since the energy expression in relativistic physics can only be accounted for, if the *resistance* of a particle to change its state of motion is due to the difference between the classical concept and the concept of photon interactions.

Let an electrostatic potential U_0 exist in the system, which is the potential difference between $x_0 = 0$ and $x_1 = L$. The classical model assumes the transfer of energy from the electrostatic field to the particle to be independent of the particle's state of motion. This is, in the interaction model, equal to no frequency changes of the photons absorbed, so the classical velocity difference Δu_n^c between two interaction processes will be:

$$E_n = \frac{m}{2} u_n^2 = \frac{n}{2} \hbar\omega_0$$

$$\Delta u_n^c = \sqrt{\frac{\hbar\omega_0}{m}} (\sqrt{n} - \sqrt{n-1}) \quad (5)$$

The velocity difference Δu_n^r due to time dilation in the electron system is significantly different:

$$\Delta u_n^r = u_n - u_{n-1} \quad (6)$$

where the velocities u_n are given by Eq. 4. The accelerations in the electrostatic field therefore cannot be independent of the particle's state of motion. If the deviation from the classical behavior, inevitably bound to occur, is interpreted as an increase of inertial mass, it must be described by a variable $\alpha'(u)$, defined by the following relations ($N\Delta x = L$):

$$\Delta u_n^c = \frac{1}{m} \frac{U_0}{N} \frac{\Delta t}{\Delta x} \quad \Delta u_n^r = \frac{1}{m\alpha'} \frac{U_0}{N} \frac{\Delta t}{\Delta x} \quad (7)$$

In addition it has to be considered that the classical theory miscalculates the already acquired velocity of the electron. The variable α' therefore has to be corrected with the ratio of velocities u_n^c/u_n^r . Then the variable α , describing the virtual change of inertia due to time dilation, can be calculated, it will be:

$$\alpha = \alpha' \frac{u_n^c}{u_n^r} = \sqrt{\frac{\hbar\omega_0}{m}} \frac{\sqrt{n} - \sqrt{n-1}}{u_n - u_{n-1}} \frac{u_n^c}{u_n^r} \quad (8)$$

The numerical calculation has been performed with an identical dataset and the result of this calculation is displayed in Fig. 2. It can be established that the effect in this case is observable, but not a real, i.e. *physical* effect. The mass of the particle remains constant over the whole range of velocities. But due to the classical conception of electrostatic interactions decreasing accelerations in the high velocity range must be interpreted in some other way: in special relativity they are interpreted as an increase of inertial mass. As this usage of the term mass is a little anachronistic, we wish to emphasize that it is to be understood in the sense of Bondi [36], who related inertia, or resistance to the total mass of a particle: "After all, the mass of a moving body can be taken to be either its rest mass or its total mass which includes the mass of its kinetic energy." While the relativistic mass formulas in Einstein's theory are "artifacts of the kinematical transformation of space and time" [14], and a result of real changes in the structure of the electron in Lorentz' theory [37], in the present context they are due to the changed characteristics of interaction described by the time dilation in moving reference frames.

The most extensive measurements of the ratio (e/m) of electrons were performed in the first fifteen years of this century by Kaufmann, Neumann, and Guye and Lavanchy [21–23]. The motivation for this extensive research was the problem of electromagnetic mass of the electron described by Abraham's theory [38]. The experiments were performed to decide between Abraham's and Einstein's or Lorentz' theory of electrons [38,12,37],

and while initial results seemed to favor Abraham, the question was finally settled by Neumann in favor of Einstein or Lorentz [22]. Neumann's result was confirmed by Guye and Lavanchy [23]. As the result obtained in the interaction model of electrostatic interaction is equal, but for numerical artifacts (in the order of 10^{-4} from 0.05 to 0.99 c , and which decrease with decreasing photon frequencies and thus smoother accelerations) to Einstein's or Lorentz results, it is fully compatible with the experimental evidence. But as this result leaves the electron energy finite even in the limit $u \rightarrow c$, it raises questions in relativistic quantum field theories which are related to the notorious infinity problems in this area.

Following Weisskopf in his treatment of the free electron [29], there are two infinite contributions to the self energy of an electron: (i) The electrostatic energy diverging with the radius a of the electron, and (ii) the energy due to vacuum fluctuations of the electromagnetic fields. The two energies W_{st} and W_{fluct} have been calculated by Weisskopf as:

$$W_{st} = \lim_{a \rightarrow 0} \frac{e^2}{a} \quad W_{fluct} = \lim_{a \rightarrow 0} \frac{e^2 \hbar}{\pi m c a^2} \quad (9)$$

As shown previously, the electrostatic contribution vanishes in the context of the present theory, because electrostatic interactions can be referred to an exchange of photons [10]: for an electron in constant motion and, especially, an electron at rest, the electrostatic fields of interaction vanish. As does the electrostatic contribution to the infinite self-energy of the electron.

In the first calculation to master the infinity problems of quantum electrodynamics Bethe derived the following expression for the Lamb shift of the hydrogen electron in an s-state [31]:

$$W'_{ns} = C \cdot \ln \frac{K}{\langle E_n - E_m \rangle_{AV}} \quad (10)$$

where C is a constant, $\langle E_n - E_m \rangle_{AV}$ the average energy difference between states m and n , and K determined by the cutoff of electromagnetic field energy. The prime refers to mass renormalization, since the - infinite - contribution to the electron energy due to electrostatic mass is subtracted. The second infinity, the infinity of vacuum fluctuations, is discarded by defining the cutoff K , which in Bethe's calculation is equal to mc^2 . But while the energy of the field could have any value, if the actual energy of the electron has a singularity at $u = c$ (and K could therefore be infinite), this is not the case if the energy remains finite in this limit: in this case the total energy difference between a relativistic electron and an electron at rest is mc^2 according to our calculations. This is, incidentally, equal to the "rest energy" of the electron. It seems, therefore, that the renormalization procedures, a common practice now for quite some time [39], may have their ultimate justification in finite electron energy as well as vanishing electrostatic energy components.

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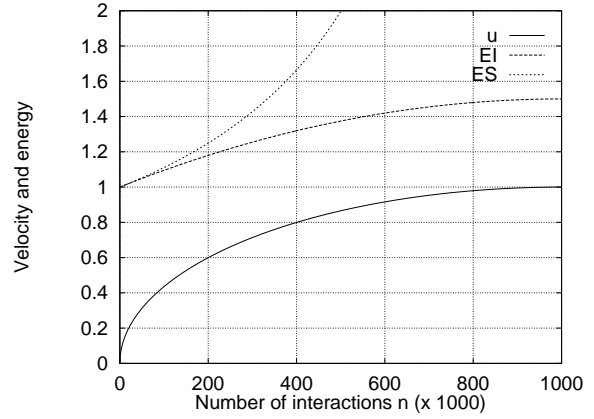


FIG. 1. Energies in the interaction (EI) and STR (ES) model. All units in the numerical calculation were relativistic units ($m = 1, c = 1, [u] = c, \hbar\omega_0 = 4 \times 10^{-6}$). The kinetic energy in the interaction model converges to a final value of $mc^2/2$, while in the STR model it becomes irregular for $u \rightarrow 1$.

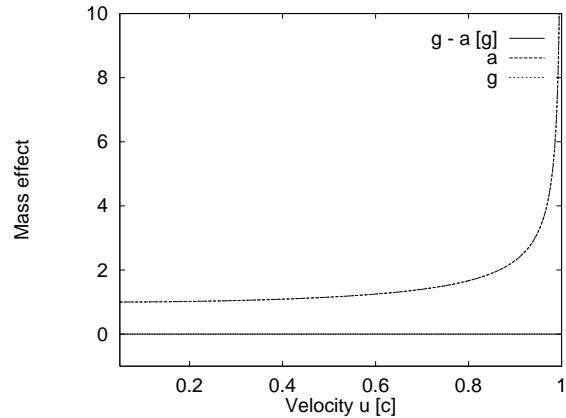


FIG. 2. Mass effect due to frequency changes of photons. The frequency shifts due to velocity of the particle lead to an observed but virtual increase of inertial mass. The difference between γ and α (g and a) is insignificant over the whole velocity range from $u = 0.05$ to $u \approx 0.99c$.